USING THE INNOVATIONS OF EXTENDED KALMAN FILTER IN TRAFFIC

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Abstract: This paper presents Divide difference filter 1st order (DD1), Divide difference filter 2nd order (DD2) and the Unscented Kalman filter (UKF) for online estimation of the traffic state model. The problem with estimate are nonlinear relation between queue length and occupation and relation between the adjacent occupancies in two adjacent arms in one intersection. This relations must be estimate simultaneously with estimate of queue. This is the main reason, why the linear Kalman filter is not sufficient.

Keywords: traffic, Kalman filter, non-linear filter

1 INTRODUCTION

In these days, the capacity of traffic network is not sufficient for increasing number of cars in majority of cities. The weak points are intersections where markedly decrease capacity of road and so it is very important to design efficient traffic control. Previously, it was used the static control of intersections where the traffic controller works with fixed time plan. Unfortunately, this type of traffic control cannot take into account the current traffic situation and change its parameters accordingly. For this reason, the dynamic control was developed because such system are able to adjust the green time, cycle time, offset etc. Now, the several types of dynamic control systems are developed and some of them are employed in some cities. The well-known are systems the TRANSYT, the SCOOT, the SCATS and the MOTION. Unfortunately, it is not possible used the same control system all over the world.

For the traffic control in Prague, it is formed dynamic traffic model (Kratochvílová and Nagy, 2004a). The principle of the control is based on estimation of the queues in the arms and on the minimalization of their total length. The queue is immeasured variable but it gives the best information about the situation on roads. In the traffic state space model, there are some unknown parameters. The value of parameters can be identify off-line by the least square method or on-line by simultaneously estimate the queue and the unknown parameters. This approach leads to the nonlinear model.

The standard KF is not suitable for estimate of nonlienar model and we needed to use innovations of extended Kalman filter (EKF). The EKF is not suitable for this model because it transform stable nonlinear relation into unstable linear relations and there is problem with compute the real derivation (it is variously). This problem some of the EKF's innovations solve. It seems that the suitable filters are the Divide difference filter 1st order (DD1), Divide difference filter 2nd order (DD2) (Norgaard *et al.*, 1998) and unscented Kalman filter (UKF) (Wan and van der Merwe, n.d.). They are besed on approximation of the nonlinear function in the state so that the Kalman technique can be used.

2 THE TRAFFIC MODEL

The traffic model is based on principle of estimation queue and their minimization. For the description of model, we need to know some parameters. The parameters can be obtained from detectors (it is the inductive loop placed in the road), the traffic controller and the contstruction of traffic networks (the roads connection, width of roads etc.).

For the model, we need to know the intensity and occupancy, which we can obtain from detectors.

- *Intensity* is the amount of passing vehicles in unit vehicles per hour [uv/h]... $I_{i,t}$.
- *Occupation* is the proportion of the sample period when the detector is being activated by vehicles [%]... $O_{i,t}$.

The traffic controler (the intersection control system) gives the additional data which are needed for the model:

- *Cycle time* of the traffic signal is a complete sequence of signal phases.
- *Green time* is the time when there is a green on one arm ... $z_{i,t}$.

The contruction dispositons determines *saturated flow*, i. e. the maximal number of cars which can go through the intersection per hour, in the case of green light being on all the time.

The immeasured value is queue and for unfortunately, it is the parameter which give us the best view of real situation on the roads.

- *Queue* is the line of vehicles waiting to proceed throught the intersection.

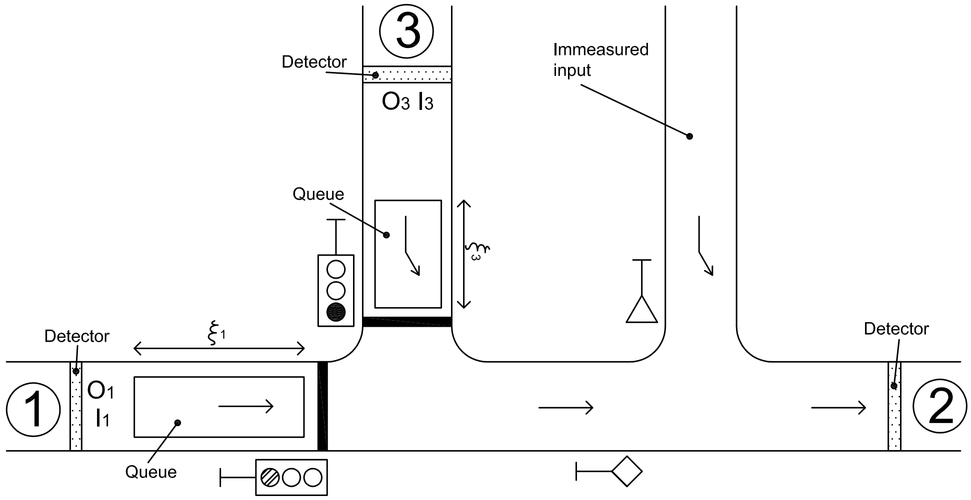

Figure 1: Three-arm one-way intersection with one immeasured input

The way to form the traffic state space model is describe in (Kratochvílová and Nagy, 2004b). In this paper, the one three-arm intersection with one immeasured input is described. The three-arm intersection is one-way and input detectors are placed on two inputs arm. The output detector is placed on 3rd arm but before them the immeasured road enters, see figure 1.

The traffic state space model for this case, it is possible to describe as

$$x_{t+1} = Ax_t + Bz_t + F + \epsilon_t \tag{1}$$
$$y_{t+1} = Cx_{t+1} + G + e_t \tag{2}$$

$$x_t = \begin{bmatrix} \xi_{1,t} & \xi_{3,t} & O_{1,t} & O_{3,t} \end{bmatrix}',$$

$$A = \begin{bmatrix} \delta_{1,t} & 0 & 0 & 0 \\ 0 & \delta_{3,t} & 0 & 0 \\ \kappa_{1,t} & 0 & \beta_{1,t} & 0 \\ 0 & \kappa_{3,t} & v_{1,3} & \beta_{3,t} \end{bmatrix}, \quad B = \begin{bmatrix} -b_1 & 0 \\ 0 & -b_3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} I_1 \\ I_3 \\ \lambda_1 \\ \lambda_3 \end{bmatrix},$$

where $b_i = (1 - \delta_{i,t})I_{i,t} - \delta_{i,t}S_i$ and $z_t = \begin{bmatrix} z_{1,t} & z_{2,t} & z_{3,t} & z_{4,t} \end{bmatrix}$.

$$C = \begin{bmatrix} -\alpha_{1,2} & -\alpha_{3,2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} \hat{\xi}_{1,t} + I_{1,t} \\ \hat{\xi}_{3,t} + I_{3,t} \\ 0 \\ 0 \end{bmatrix}.$$

We have two types of state and it is given by parameter δ which indicates the queue. Practically, it means that when the queue exist, the output intensity is the capacity (saturation flow times relative green time). When the queue not exist, the output intensity is the input one limited by green time too. In case that the queue exists $\delta = 1$ and if not $\delta = 0$. In the equation of occupancy are used the relation between occupancy and queue length in individual arm, parameters κ,β, λ, and relation between the adjacent occupancies in two adjacent arms in one intersection, parameter v.

3 THE NONLINEAR FILTERS

3.1 *The DD1 Filter and DD2 Filter*

The nonlinear estimation via the DD1 or DD2 filter is based on Taylor approximations. The DD1 and DD2 filters works similarly, but the DD1 filter is based on first order approximation, it means the derivatives of first order, and DD2 filter replace the derivate first and second order by divided differences. The filter works with general nonlinear model of a dynamic system

$$\begin{aligned} x_{t+1} &= f(x_t, u_t, w_t), \\ y_t &= h(x_t, v_t), \end{aligned}$$

where:

w_t is process noise,

v_t is measurement noise.

The DD1 and DD2 filter in principle corresponds to the EKF except that the Jacobians are replaced by divided differences. The state update is therefore the same as in the EKF. The differenci is alone found in the update of the various covariance matrices.

In the DD1 and DD2 filter, algorithms are compute from the filtrations and predictions covariances matrices instead of the all matrices which are decomposed by Cholesky factors . The compute from filtrations and predictions covariances matrices has the advantage in much smaller compute demands. One provided is to update the Cholesky factors of the covariance matrices directly. The whole algorithm is in (Duník, 2005).

For using the filters for estimate of parameters, it is needed extended the state about parameters, in our case about κ, β, λ and v. The parameters are than estimate and update in each step.

3.2 *The Unscented Kalman Filter*

In the EKF is state approximate according to Gaussian distribution and it means, that the nonlinear system will be linear. The Unscented Kalman Filter (UKF) solves this problem using deterministic sampling approach. The state is approximate by a Gaussian distribution too but it is used the minimal set of points which are select according to given rules. The complicatedness of compute the UKF is the same as by the EKF.

The UKF is based on the unscented transformations (UT). This is a method for calculating the statistic of random variable which are nonlinear. The time update and measurement update equations and each derivation are in (van der Merwe, 2004).

For using UKF, it is needed the right select of coefficient κ which must give with the state dimension the value 3. This is main problem which can we have with using this filter in traffic. When we have, for example, two four-arm intersection, our state dimension is 48. In current, the new updates of the UKF are created and this version removes this problem and is more stable.

4 EXPERIMENTS

The experiments was made on three-arm intersection with immeasured input. The intersection is not real but to the input parameters as intensity and occupancy was used the data from real intersection. The queue length was simulated. Our preconditions are that the input intensity from arm with detectors is bigger than intensity from immeasured arm.

In this experiments, the KF and DD1 filter are comparison according to criterion - the best estimate of queue with respect to the real queue. It was used the data from 3 days such as there was the long queue. The queue was estimated with KF at first and the estimate is on figure 2. The queue estimate is poor because the KF was not managed the quick changes. On the other hand, the filter DD1 did not have the problem with this, see figure 3.

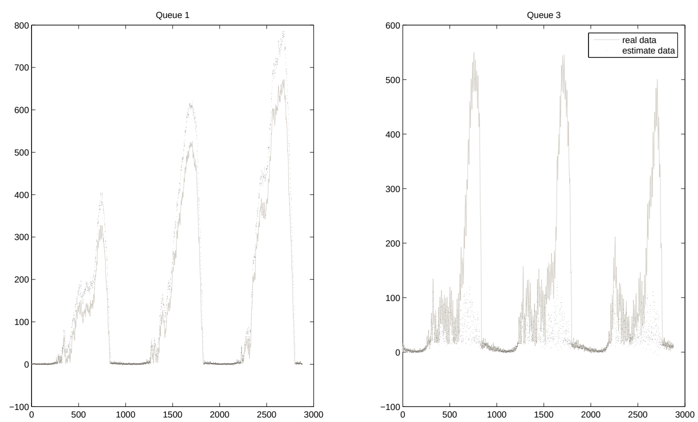

Figure 2: The true queue lengths and their KF estimates

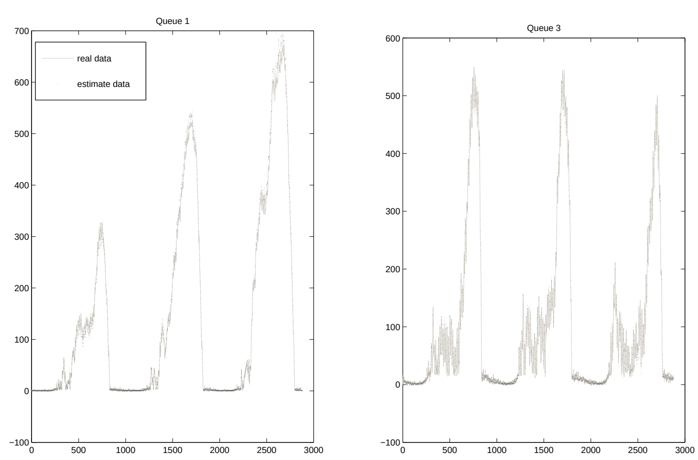

Figure 3: The true queue lengths and their DD1 filter estimates

The total results, from this experiment, are in tabular 1. The deviation is the sum of absolute differences between true and estimate queue. The column Improvement shows, the improvement of estimate when instead of KF was used the DD1 filter. From this comparison, it is seen than the DD1 filter is much better than KF. The estimate of queue via DD2 filter and UKF was similar the DD1 filter estimates but the compute was slower.

Table 1: The comparison of estimate via KF and DD1 filter

Arm No.	True queue lengths [m]	KF deviation [m]	KF in %	DD1 filter deviation [m]	DD1 filter in %	Improvement in %
1	$4,113 \cdot 10^5$	$9,986 \cdot 10^4$	24,28	$2,017 \cdot 10^4$	4,90	$+19,38$
3	$2,934 \cdot 10^5$	$2,101 \cdot 10^5$	71,62	$1,275 \cdot 10^4$	4,35	$+67,27$
1+3	$7,047 \cdot 10^5$	$3,100 \cdot 10^5$	43,99	$3,293 \cdot 10^4$	4,67	$+39,32$

5 CONCLUSION

In this paper, it was described tree filters for nonlinear state estimation of traffic model. The filters was comparison with linear KF on queue estimate of three-arm intersection. The several experiments shown that the linear Kalman filter has problem with unknown input road and with inaccurate a priori parameters of relation between queue and occupancy. The assumption of stationarity of model parameters prevents KF from tracking rapid changes of the intensity.

In future, it is needed to tested the nonlinear filters on bigger micro-regions. The micro-regions can be formed from different types of intersections (controlled and uncontrolled) and can be connected to the loops.

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